Teacher: SB (9476296853) BMG2CC1B

Solve

$yzp + 2xq = xy$

Ans:

Given

$$yzp + 2xq = xy$$

Lagrange's Auxiliary Equation is.

$$\frac{dx}{yz} = \frac{dy}{2x} = \frac{dz}{xy} \quad \text{---}(1)$$

So we have from (1)

$$\cancel{\frac{dx}{yz} = \frac{dy}{2x}}$$

$$\frac{dx}{yz} = \frac{dz}{xy} \Rightarrow x\,dx = z\,dz$$

$$\Rightarrow \frac{x^2}{2} = \frac{z^2}{2} + \frac{c_1}{2} \quad \text{(integrating)}$$

$$\Rightarrow x^2 - z^2 = c_1$$

c_1 = constant of integration

We again have ~~from~~ from (1)

$$\frac{dy}{2x} = \frac{dz}{xy}$$

$$\Rightarrow y\,dy = 2\,dz$$

$$\Rightarrow \frac{y^2}{2} = 2z + \frac{c_2}{2} \quad \text{[By integration]}$$

$$\Rightarrow y^2 - 4z = c_2$$

c_2 = constant of integration.

Hence ~~a~~ general solution is

$$x^2 - y^2 = \phi\,(y^2 - 4z)$$

where ϕ is an arbitrary function.

Solve: $yp + xq = xy$

Given $yp + xq = xy$ — (1.)

Comparing with Lagrange's ~~Auxiliary~~ Equation $Pp + Qq = R$ we have Lagrange's auxiliary equation as.

$$\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xy} \quad -(1.)$$

Now from (1.)

$$\frac{dx}{y} = \frac{dy}{x} \Rightarrow xdx = ydy$$

By integration we have.

$$\frac{x^2}{2} = \frac{y^2}{2} + \frac{c_1}{2}$$

$$\Rightarrow x^2 - y^2 = c_1$$

c_1 = Integrating constant.

Again from (1)

$$\frac{dy}{x} = \frac{dz}{xy} \Rightarrow ydy = dz$$

$$\Rightarrow \int ydy = \int dz + \frac{c_2}{2}$$

$$\Rightarrow y^2/2 = z + c_2/2$$

$$\Rightarrow y^2 - 2z = c_2$$

c_2 = constant of integration.

Hence the general solution

$$\phi(x^2 - y^2, y^2 - 2z) = 0$$

where ϕ is an arbitrary function.